MSE 301 Integrated Computational Materials Engineering
Homework 4: Finite Element Method
Due in class on Tuesday May 07

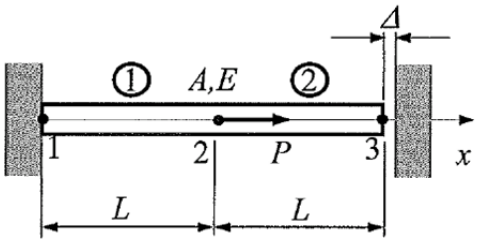
A. Conceptual Short Answer Questions: (20 points, 5 points each)

1. What are differences between strong and weak from of a partial differential equation (PDE)?
2. What are the conditions in choosing an approximate solution for a PDE?
3. What are the factors that determine the accuracy of the finite element solution of a PDE? Compare 1D to 2D or 3D problems.
4. What are the factors that determine the computational efficiency of a finite element algorithm?

B. Calculation: (60 points total)

1. Determine the support reaction forces at the two ends of the bar shown below. (20 points)

$P = 6.0\times10^4\ \mathrm{N}, \quad E = 2.0\times10^4\ \mathrm{N/mm^2},$
$A = 250\,\mathrm{mm^2}, \quad L = 150\,\mathrm{mm}, \quad \Delta = 1.2\ \mathrm{mm}$



2. Construct the week forms:

a) A non-linear equation (20 points)

$$-\frac{d}{dx}\left(u\frac{du}{dx}\right)+f=0, \quad 0<x<1$$

$$\left(u\frac{du}{dx}\right)_{x=0}=0, \quad and \quad u(1)=\sqrt{2}$$

b) Two-dimensional steady-state flow of viscous, incompressible fluids (stream function-vorticity formulation; $\psi(x,y)$, $\xi(x,y)$): (20 points)

$$\left.\begin{aligned} -\nabla^2\psi - \xi &= 0 \\ -\nabla^2\xi + \frac{\partial\psi}{\partial x}\frac{\partial\xi}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial\xi}{\partial x} &= 0 \end{aligned}\right\} \quad in\ \Omega$$

Assume all the essential boundary conditions are specified to be zero.

C. <u>Simulations:</u> (20 points)

Modify the provided Matlab code to solve the following BVP's using 5 and 10 elements:

a) $$u'' = 1 \quad 0 < x < 3$$
$$u(0) = 0, u(3) = 0$$

b) $$u'' + u' - 2u = 0 \quad 0 < x < 1$$
$$u(0) = 0, u(1) = 1$$

Print the calculated matrix and vectors, and plot the results for both cases against the exact solution for each case.